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## CHALLENGES FOR SUPERSTRING COSMOLOGY

RAM BRUSTEIN AND PAUL J. STEINHARDT

Department of Physics,  
University of Pennsylvania, Philadelphia, PA 19104

### ABSTRACT

We consider whether current notions about superstring theory below the Planck scale are compatible with cosmology. We find that the anticipated form for the dilaton interaction creates a serious roadblock for inflation and makes it unlikely that the universe ever reaches a state with zero cosmological constant and time-independent gravitational constant.

Superstring theories have been extensively studied as models of unified theories [1]. Effective field theories in four dimensions are used to describe string theory below the Planck scale. There are strong prejudices about the form that the four dimensional low energy effective field theory derived from superstrings must take in order for superstrings to be a viable model of particle interactions. In this paper, we extract the most generic elements of the “current superstring lore” (CSSL) and examine their implications for the early evolution of the universe.

Our goal here is to be strict and systematic in bringing CSSL and cosmology together. Hence, it is necessary to carefully lay down the components of CSSL and to not deviate from them. In the past, several authors have encountered some of the cosmological problems of superstrings outlined below and have resorted to adding new terms in the effective potential to solve them [2]-[5]. However, these added terms violate one or more key tenets of present superstrings lore; the cosmology problems of CSSL are thereby masked. The main components of CSSL are:

(1) The effective theory below the Planck scale is described by a weakly-coupled, four-dimensional,  $N = 1$  supergravity field theory. Hence, the effective action is Kahler invariant, described by a real Kahler potential,  $K(\phi_i)$ , and a holomorphic superpotential  $W(\phi_i)$ :

$$\mathcal{L}_{\text{eff}} = \sum_{i,j} \frac{1}{2} K_{\phi_i, \bar{\phi}_j} \nabla \phi_i \cdot \nabla \bar{\phi}_j - e^K \left[ \sum_{i,j} K^{\phi_i \bar{\phi}_j} (D_{\phi_i} W)(D_{\bar{\phi}_j} W)^\dagger - 3|W|^2 \right] \quad (1)$$

where the sum is over (complex-valued) chiral superfields fields  $\phi_i$ , and  $D_{\phi_i} = \partial_{\phi_i} - K_{\phi_i}$ ,  $K_{\phi_i} = \partial_{\phi_i} K$  and  $K^{\phi_i \bar{\phi}_j} = K_{\phi_i \bar{\phi}_j}^{-1}$ . The Kahler potential,

$$K = -\ln (S + S^*) - 3\ln (T + T^*) + \dots \quad (2)$$

includes two fields:  $S$  (related to the dilaton) and  $T$  (related to the breathing

mode). According to CSSL, the contributions of other moduli (fields similar to  $T$ ), matter fields and higher order corrections not shown explicitly here can be expressed as corrections to  $K$  and  $W$ .

(2) The unified gauge coupling constant is fixed at small value  $g_{GUT} \approx .7$  to agree with recent hints from LEP suggesting that, within the minimal supersymmetric standard model, a unification of coupling constants occurs at a scale  $M_{GUT} \approx 10^{16}$  GeV. The gauge coupling constant is determined by the expectation value of the real part of  $S$ ,  $\langle \text{Re} S \rangle \sim \frac{1}{g^2} > 1$ .

(3) The source of dilaton interactions is in the hidden sector of the theory and appears at some intermediate scale  $\Lambda_c \approx 10^{14}$  GeV. The dilaton and moduli interactions induce supersymmetry breaking in the observed sector of the theory [6]. Supersymmetry breaking in the observed sector occurs at approximately the electroweak scale  $10^3$  GeV. The hierarchy between the scale  $\Lambda_c$  of the hidden sector and supersymmetry breaking in the observed sector, is created because of the weakness of the induced dilaton interactions.

(4) The dilaton potential is set by non-perturbative interactions. An important consequence of Condition (3) above is that the dilaton potential vanishes at temperatures between the Planck scale ( $10^{19}$  GeV) and the intermediate scale,  $\Lambda_c \approx 10^{14}$  GeV. Below  $\Lambda_c$ , the superpotential is of the form:

$$W(S, T) = \sum_j e^{-\alpha_j S} f_j(S, T, \dots), \quad (3)$$

where  $\alpha_j$  are constants and  $f_j$  depend on matter fields, and may contain powers of  $S$ . The potential is, using Eq. (1):

$$V(S) = (S + S^*) |\partial_S W(S)|^2 - \frac{1}{S + S^*} W(S)^2 - \frac{3}{S + S^*} |W(S)|^2 \quad (4)$$

The dependence on  $T$  and other moduli is not shown explicitly here because their expectation values are fixed by modular invariance [7].

Although some of our arguments depend only on the general, non-perturbative form of the superpotential, for concreteness we will assume the current leading candidate as a source of this non-perturbative potential: gaugino condensation [8],[9]. Above the scale of gaugino condensation,  $\Lambda_c$ , the superpotential is identically zero. Below  $\Lambda_c$ , the superpotential has the form given in Eq. (3). In this case,  $\alpha_j = 3k/2\beta$ , where  $k$  is a positive integer,  $\beta$  is determined by the one-loop beta-function of the hidden gauge group, and the  $f_j$ 's are known constants, determined by the structure of the hidden gauge group and the potential for the  $T$  field and matter fields [10].

$$W(S) = \sum_j a_j e^{-\alpha_j S} \quad (5)$$

If the gauge group is simple there is only one term in the sum. If the gauge group is semi-simple, each additional term corresponds to gaugino condensation in one of the distinct factors of the group. The rank of the gauge group is restricted to be at most 18. The sum in Eq. (5) is therefore a small finite sum.

Now we wish to show how this prevalent view of superstrings runs into immediate problems once cosmology is considered. First, we shall argue how dilaton interactions create a serious roadblock for inflationary cosmology. Since inflation is not yet a proven idea, this problem is arguably the least severe. Then, we shall see how the anticipated dilaton interaction makes it highly unlikely that the universe ever reaches a state with zero (or small) cosmological constant and a state in which Newton's constant  $G$  is time-independent,  $\dot{G} = 0$ . Here, the contradictions are with strong observational limits; e.g., see [11]-[14]. In the end, we do not claim a rigorous theorem. However, we have been unable to identify a viable model, and we seem to be driven to unattractive extremes (at best).

Any type of inflationary scenario requires an epoch in which the false vacuum (potential) energy density dominates the energy density of the universe [15]. The false vacuum energy density must drive the scale factor  $a(t)$  to grow as  $a(t) \propto t^m$  where  $m \gtrsim 4$  [16] in order to solve the cosmological flatness, horizon, and monopole problems. The anticipated dilaton interaction can prevent the universe from undergoing superluminal expansion in two ways. Before the dilaton settles into a stable minimum of its potential, the dilaton kinetic energy density can dominate the vacuum energy density. Once the dilaton settles at a minimum and its kinetic energy becomes negligible, the problem may be that there is not sufficiently large potential energy density to drive inflation.

Let us first consider the problem before the dilaton settles to a minimum. Numerical integration is required to obtain quantitative results. However, there is a useful analogy to Brans-Dicke theory that allows us to explain the essence of our results in a simple way. Both a Brans-Dicke scalar and the dilaton are non-minimally coupled to the scalar curvature. Through its non-minimal coupling, the Brans-Dicke field  $\phi$  is subject to a force proportional to the false vacuum energy density. Instead of the vacuum energy density being focused totally into inflating the scale factor  $a(t)$ , some of it is funneled off into driving the  $\phi$  field [17]. The kinetic energy density of the  $\phi$  field changes the overall equation of state, slowing down the expansion rate (i.e., reducing  $m$ ).

If the Brans-Dicke field is Weyl transformed so that the scalar curvature term assumes standard Einstein form  $((16\pi M_{Pl}^2)^{-1}\mathcal{R})$ ,  $\phi$  has canonical kinetic energy density, but the false vacuum energy density  $V_F$  is modified by  $\phi$ -dependent factor,  $V(\phi) = V_F \exp(-\beta\phi/M_{Pl})$ , where  $\beta = \sqrt{64\pi/(2\omega + 3)}$  and  $\omega$  is the Brans-Dicke parameter that determines the deviation from Einstein

gravity. The equations of motion for the Weyl-transformed theory are:

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV(\phi)}{d\phi}, \quad (6)$$

where  $H^2 \equiv (\dot{a}/a)^2 = 8\pi(\frac{1}{2}\dot{\phi}^2 + V)/(3M_{Pl}^2)$ . The solution is  $a(t) \propto t^{(2\omega+3)/4}$  so that  $\omega \geq 5$  is required to resolve the horizon and flatness problems. A slightly weaker bound  $\omega > 1/2$  is required to have any superluminal expansion [17]. For  $\omega < 1/2$ , the potential for  $\phi$  in the Weyl transformed theory is so steep that the  $\phi$  kinetic energy density grows to dominate any potential energy density.

In superstrings, the dilaton  $\phi$  is related to  $S$  by  $Re S = e^\phi$ . The effective potential is identically zero until some intermediate scale  $\Lambda_c \approx 10^{14}$  GeV. Since inflation requires a non-zero vacuum energy density, inflation can only occur after a non-zero potential is generated below  $\Lambda_c$ . Below  $\Lambda_c$ , the dilaton potential assumes the form given in Eq. (5). Any prospective inflaton must couple to  $S$ . Under the assumption that the dilaton potential is strictly non-perturbative, the false vacuum energy must depend on  $S$  through one or more terms in Eq. (5). Hence, the following argument is independent of the type of inflationary model (e.g., chaotic, extended, etc.).

In Figure 1, we plot some typical dilaton potentials corresponding to one and two gaugino condensates. Note that the potential is unbounded below for  $\phi < 0$ ; this corresponds to the strong coupling region, where the non-perturbative analysis that led to this potential cannot be trusted. Hence, we will restrict ourselves to  $\phi > 0$ . The striking feature of the potential is its steepness. Because the potential is non-perturbative in origin, the potential drops as  $e^{-2\alpha S} \sim e^{-2\alpha e^\phi}$ , for large  $\phi$ , where  $\alpha$  is one of the coefficients in Eq. (5).

Figure 1. Non Perturbative Dilaton Potential  $V(\phi)/M_{Pl}^4$ . The dashed line corresponds to one gaugino condensate and the solid line corresponds to two gaugino condensates. The insert shows  $V(\phi)$  in the region where the potential resulting from two gaugino condensates has a minimum, scaled to show the minimum and the steepness of the potential.

The precise value of  $\alpha$  depends on the hidden gauge group, but among the known examples we find  $\alpha \geq 24\pi^2/r$ , where  $r$  is an integer number of order  $\mathcal{O}(10)$ . Expanded about any  $\phi_0 > 0$  ( $S_0 \equiv e^{\phi_0} > 1$ ), the potential can be approximated by  $\exp(-2\alpha S_0(\phi - \phi_0))$ . Comparing this with the Brans-Dicke theory, we find a small effective  $\omega \approx 4.5 \times 10^{-4}(r/S_0)^2 - 1.5$ . For typical hidden sector groups,  $\omega < 1/2$ , a value too small for inflation owing to the steepness of the potential; for  $E_8$ , we find the maximal value  $r = 90$  which, combined with  $S_0 = 1$  squeaks past superluminal expansion with  $\omega < 2$ , but still  $\omega$  is too small to solve the cosmological horizon and flatness problems.

Consequently, *until  $\phi$  settles near a minimum of its potential, the dilaton kinetic energy totally dominates the potential energy density, thereby blocking inflation of any kind.*<sup>1</sup>

The best-hope for inflation then becomes a scenario in which the dilaton first rolls to a minimum, and then inflation is driven by other fields. There are two problems with this approach, though. First, as is apparent from Fig. 1, for  $\phi$  be trapped in any minimum some very strict constraints on the initial conditions of  $\phi$  have to be satisfied. Initially (above  $\Lambda_c$ ) there is no potential and it seems reasonable to assume that  $\phi$  is has an arbitrary value that is spatially varying. If  $\phi$  begins to the right of the “bumps” in the potential, it rolls continuously towards  $\phi \rightarrow \infty$ . If  $\phi$  begins close to  $\phi = 0$ , the potential is so steep that  $\phi$  rolls right over the bumps and continues onto  $\phi \rightarrow \infty$ . Only for a narrow region about the minimum will  $\phi$  be properly trapped. However, this condition is not sufficient: the energy density must be positive to have inflation, whereas the example in Fig. 1 has a minimum with negative vacuum energy density. The minimum must also be *metastable* since, after inflation, the universe needs to find its way down to a lower energy state with zero cosmological constant. Thus far, we have been unable to construct a superpotential of the form in Eq. (5) which has the desired properties.

Even without inflation, we have the embarrassing problem of understanding why  $\phi$  gets trapped at a minimum rather than rolling past the bumps and evolving forever. The dilaton must be trapped because, otherwise, its evolution leads to an unacceptably large time-variation of Newton’s constant  $G$  [13],[14]. As we have argued, so long as  $\phi$  is unpinned, the effective  $\omega$  is

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<sup>1</sup>Strictly speaking, there can be inflation if  $\phi$  happens to begin very close to the peak of one of the small hills, e.g., see the two-gaugino potential in Fig. 1 . However, this seems to be an extraordinarily fine-tuned and unattractive initial condition.



less than unity, whereas cosmology demands  $\omega$  to be greater than 50 in order to meet the constraints of primordial nucleosynthesis and solar system tests (e.g., Viking radar-ranging) push the constraints even higher,  $\omega > 500$  [12]. At this point, the only means of trapping  $\phi$  is if its initial value is close to one of the minima in the first place. It would be nice if some natural damping mechanism existed which forces the dilaton to settle in its shallow minimum.

But is the minimum of a dilaton potential where the universe really ought to be? The problem here is that it is very difficult<sup>2</sup> to avoid a minimum with negative cosmological constant! This was already a problem in generic supergravity theories [19]; the problem is exacerbated when one considers the more special non-perturbative potential in Eq. (5).

A minimum of the potential satisfies  $\partial_S V(S) = 0$  and can be supersymmetric in the  $S$  sector,  $D_S W = 0$ , or produce supersymmetry breaking,  $D_S W \neq 0$ , where  $D$  is the Kahler derivative defined below Eq. (3). It is straightforward to show that any extremum of  $V(S)$  in the weak coupling region (namely,  $ReS > 1$ , where the desirable minimum should be according to condition 2 of the CSSL), with  $W(S)$  of the form in Eq. (5) and  $D_S W \neq 0$  is a saddle point of  $V(S)$ , rather than a stable minimum [10]. Hence, the present universe could not be in a vacuum state which is supersymmetry breaking in the  $S$  sector.

The only remaining possibility is that the present state of the universe corresponds to a minimum with  $D_S W = 0$ : either  $\partial_S W(S)|_{S_{min}} = W(S_{min}) = 0$  (a minimum of Type A); or  $W(S_{min})$  and  $\partial_S W(S)|_{S_{min}}$  are both non-zero

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<sup>2</sup>A special mechanism for obtaining zero cosmological constant [11],[18] has been used in the literature [8],[2] in which a field appears in the Kahler potential but not in the superpotential. This approach necessarily leads to a flat direction in the potential and a massless field. It is unknown if there is a mechanism to give the field a mass and yet maintain zero cosmological constant.

(a minimum of Type B). If supersymmetry were unbroken in the whole theory then the potential in Eq.(4) is exactly correct and because  $D_S W = 0$  but  $W(S_{min}) \neq 0$  in Type B,  $V(S_{min})$  is negative (see Eq. (4)) and Type B minima would necessarily have an unacceptable negative cosmological constant. However, supersymmetry has to be broken in some sector the theory. This means that the factor 3, in Eq.(4) gets modified to a smaller number  $n < 3$ . In all presently known examples, supersymmetry breaking occurs in the moduli sector [7],[10] and  $n > 1$  always. Provided  $n > 1$ , the conclusion that the potential is negative at Type B minima is not modified.

The last hope for a realistic vacuum state is a minimum of Type A. However, we shall now argue that even if one is able to arrange a minimum with zero cosmological constant of Type A, it is almost always accompanied by a lower energy minimum of Type B with negative cosmological constant in which the universe is more likely to be trapped. This is the situation depicted in Figure 2, it corresponds to an example found in ref.[20]. We ignore here contributions from supersymmetry breaking in other sectors of the theory.

Figure 2. Dilaton potential corresponding to gaugino condensation in the hidden group  $G = SU(2)_{k=1} \times \left[ SU(2)_{k=2} \right]^4 \times \left[ SU(2)_{k=3} \right]^4$ .

First, let us consider the case where the coefficients in Eq. (5) are all real and there is a minimum of Type A for some real value of  $S = S_A$ . For any

hidden gauge group, the coefficients  $\alpha_j$  in Eq. (5) can all be written as  $n_j\tilde{\alpha}$ , where the  $n_j$ 's are integers. Hence, if we set  $z = e^{-\tilde{\alpha}S}$ ,  $W(S)$  becomes a polynomial  $P(z)$ . As  $S \rightarrow \infty$ ,  $W(S) \rightarrow 0$  exponentially for superpotentials of non-perturbative form; consequently,  $P(z=0) = 0$ . If  $z_A = e^{-\tilde{\alpha}S_A}$  (which is real by our assumptions), then, since  $W(S_A) = \partial_S W(S_A) = 0$ , we can write  $P(z) = z(z - z_A)^2 \tilde{P}(z)$ . There may be many Type A minima along the real axis, but let us assume that  $S_A$  is the most positive value. Now let us consider  $P(z)$  between  $z = z_A$  and  $z = 0$  (corresponding to  $S_A < S < \infty$ ). Rolle's theorem says that between every two zeros of a real function lies a zero of its derivative. Since  $P(z)$  has a zeros at  $z = z_A$  and  $z = 0$ , this means that whenever we have a minimum of Type A, there will be a point between, call it  $z'$ , corresponding to  $S' > S_A$ , where  $P'(z') = 0$  or  $\partial_S(W) = 0$ . If we also had  $W = 0$  and this were a minimum, then it would be of Type A, contradicting our assumption that  $S_A$  is the most positive minimum of Type A. That leaves two possibilities: (i)  $W \neq 0$ , in which case  $V(S') < 0$  (see Eq. (4); or (ii)  $W = 0$ , and, hence,  $V(S') = 0$ , but  $S'$  is not a local minimum. In either case, we see that *there must be a range of the effective potential with negative  $V(S)$ , in the neighborhood of  $S'$* . This means that the global minimum of  $V(S)$  is negative. In practice, these are relatively deep minima which the universe can avoid only for special initial conditions.

If we consider complex coefficients in Eq. (2), there is a useful, albeit somewhat weaker generalization of Rolle's theorem that applies specifically to polynomial  $P(z)$  [21]. We begin with the same assumptions that  $P(z)$  has a zero at  $z = 0$  ( $S \rightarrow \infty$ ) with multiplicity  $k_0 \geq 1$  and a multiplicity  $k_A \geq 2$  zero (Type A minimum) at some real  $z = z_A$ . Then, there is at least one additional  $z = z'$  in the complex plane for which  $P'(z') = 0$ . The point  $z'$  lies within a bounded region  $\mathcal{A}$  about the segment  $\overline{OA}$  which joins

$z_0$  and  $z_A$ . The same reasoning as above implies that there is a neighborhood of  $z'$  where  $V(S) < 0$ , a potentially dangerous region of the potential with negative cosmological constant. Region  $\mathcal{A}$  for polynomial  $P(z)$  of order  $n$  is the union of the two circles for which  $\overline{OA}$  is a chord subtending an angle  $2\pi/(n+1-k_0-k_A)$ . For fixed  $k_0$  and  $k_A$ ,  $\mathcal{A}$  grows as  $n$  increases. For concreteness, let us consider the minimal case,  $k_0 = 1$  and  $k_A = 2$ . Then, for  $n \leq 4$ ,  $\mathcal{A}$  lies totally in the physically allowable region  $\text{Re } S > 0$  ( $|z| < 1$ ); in this case, the region of  $V(S)$  with negative cosmological constant near  $z'$  is a real danger. However, for sufficiently large  $n > 4$  (depending on the value of  $z_A$ ),  $\mathcal{A}$  extends to the unphysical region  $\text{Re } S < 0$  ( $|z| > 1$ ). Hence, for sufficiently large  $n$ , it is conceivable that  $V(S)$  only has Type A minima in the physical region. So, the situation is not completely hopeless, but one must pay a heavy price: (a) the coefficients  $a_j$  in Eq. (5) must be complex, or equivalently,  $\langle \text{Im} S \rangle \neq 0$ , which may not be possible for realistic CP invariant theories; and (b) in order to satisfy  $n > 5$  for  $k_0 = 1$  and  $k_A = 2$  (or, more generally,  $n > 2 + k_0 + k_A$ ) so that the Type B minima are pushed into the unphysical region, at least five gaugino condensates are required. This combination of conditions seems unwieldy, unlikely and unattractive.

We are, therefore, forced to conclude that it seems difficult to construct a model consistent with current superstring lore which can support inflation or which can lead to a vacuum state with zero cosmological constant and time-invariant Newton's constant. While we have pointed out some exotic loopholes entailing special initial conditions and complex superpotentials, we would recommend against this unattractive approach. Rather, we have presented this discussion because we believe that superstring theorists should be aware of the impending cosmological disasters and take up our challenge: Is there a plausible modification of superstring lore that will fit better with

cosmology, and does this modification give us a promising implications for particle physics?

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